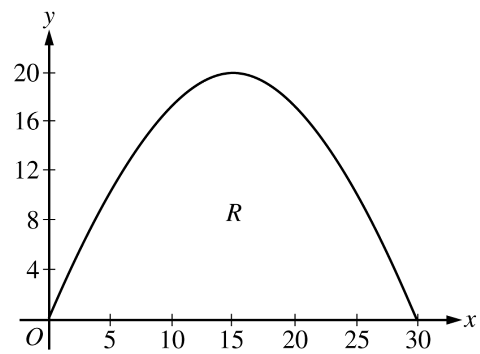
AP® CALCULUS BC
2009 SCORING GUIDELINES (Form B)

Question 1

A baker is creating a birthday cake. The base of the cake is the region R in the first quadrant under the graph of $y = f(x)$ for $0 \le x \le 30$, where $f(x) = 20\sin\left(\frac{\pi x}{30}\right)$. Both x and y are measured in centimeters. The region R is shown in the figure above. The derivative of f is $f'(x) = \frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)$.



(a) The region R is cut out of a 30-centimeter-by-20-centimeter rectangular sheet of cardboard, and the remaining cardboard is discarded. Find the area of the discarded cardboard.

(b) The cake is a solid with base R. Cross sections of the cake perpendicular to the x-axis are semicircles. If the baker uses 0.05 gram of unsweetened chocolate for each cubic centimeter of cake, how many grams of unsweetened chocolate will be in the cake?

(c) Find the perimeter of the base of the cake.

(a) Area $= 30 \cdot 20 - \int_0^{30} f(x)\,dx = 218.028\ \text{cm}^2$	$3: \begin{cases} 2: \text{integral} \\ 1: \text{answer} \end{cases}$
(b) Volume $= \int_0^{30} \frac{\pi}{2}\left(\frac{f(x)}{2}\right)^2 dx = 2356.194\ \text{cm}^3$ Therefore, the baker needs $2356.194 \times 0.05 = 117.809$ or 117.810 grams of chocolate.	$3: \begin{cases} 2: \text{integral} \\ 1: \text{answer} \end{cases}$
(c) Perimeter $= 30 + \int_0^{30} \sqrt{1 + (f'(x))^2}\,dx = 81.803$ or 81.804 cm	$3: \begin{cases} 2: \text{integral} \\ 1: \text{answer} \end{cases}$

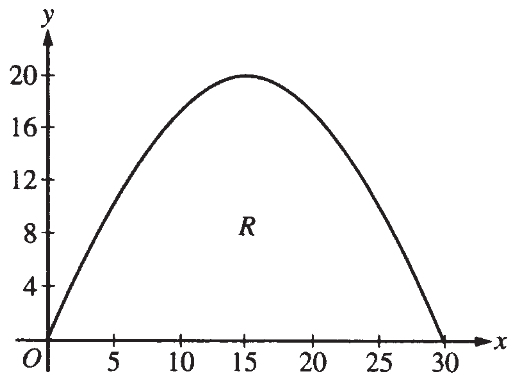
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

Area of R = $\int_0^{30} 20\sin\left(\frac{\pi x}{30}\right)dx \approx 381.972$

remaining cardboard = $30 \times 20 - R$

$\doteqdot 218.028 \text{ cm}^2$

Work for problem 1(b)

area of semicircle = $\frac{1}{2}r^2\pi$

$r = \frac{1}{2}4$ $\quad\therefore$ area of semicircle = $\frac{1}{8}4^2\pi$

$\therefore$ Volume = $\frac{1}{8}\pi \int_0^{30} \left(20\sin\left(\frac{\pi x}{30}\right)\right)^2 dx$

$\doteqdot 2356.19449 \text{ cm}^3$

amount of chocolate = $0.05 \times 2356.19449 = 117.8097$ g

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Continue problem 1 on page 5.

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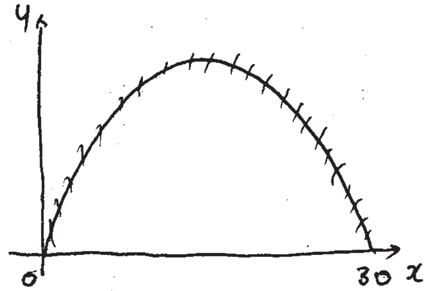
Work for problem 1(c)



perimeter = shaded line + ~~#~~ ~~x~~ a portion of x-axis

→ a portion of x-axis = 30. cm

$$\text{shaded line} = \int_0^{30} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^{30} \sqrt{1+\left(\frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)\right)^2}\, dx$$

$$\approx 51.80370374 \text{ cm}$$

shaded line + a portion of x-axis

$$= 81.80370374 \text{ cm}$$

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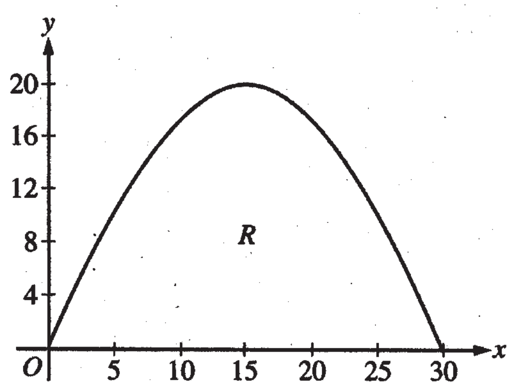
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

Area of original cardboard : $30 \times 20 = 600 \text{ cm}^2$ — ①

Area of $R = \int_0^{30} f(x)\,dx = \left[-\frac{600}{\pi}\cos\left(\frac{\pi x}{30}\right)\right]_0^{30} = 381.972 \text{ cm}^2$ — ②

Area of discarded cardboard : ① − ② $= 600 - 381.972 = 218.028 \text{ cm}^2$

Note that R does not go beyond the height (y-value) of 20, making the above calculation valid.

Work for problem 1(b)

Area of a semi-circle with radius $r = \frac{1}{2}\pi r^2$, thus area of a cross-section of R = $\frac{1}{2}\left(\frac{1}{2}\cdot 20\sin\left(\frac{\pi x}{30}\right)\right)^2\pi$. Integrating this from x=0 to 30, we get

$\frac{\pi}{8}\int_0^{30}\left(20\sin\left(\frac{\pi x}{30}\right)\right)^2 dx$, we get 2356.194 cm^3 of cake.

Each cubic centimeter of the cake has 0.05 gram of chocolate; so

$2356.194 \text{ cm}^3 \cdot 0.05 \text{ g/cm}^3 = 117.810$ g. of chocolate will be in the cake.

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Continue problem 1 on page 5.

Work for problem 1(c)

The perimeter of the base of the cake can be divided into two parts.
The straight component on the x-axis is one, with length $30-0=30$cm.
The curve of $f(x)$ from $x=0$ to 30 is the other.
The length of a curve is given by $\int_a^b \left(1+\left(\frac{dx}{dy}\right)^2\right)dx$.
$f'(x)$ is given by the question, thus the curve has length
$\int_0^{30}\left(1+\left(\frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)\right)^2\right)dx = 95.797$ cm.
Adding both components, the perimeter of $R = 30+95.797 = 125.797$cm.

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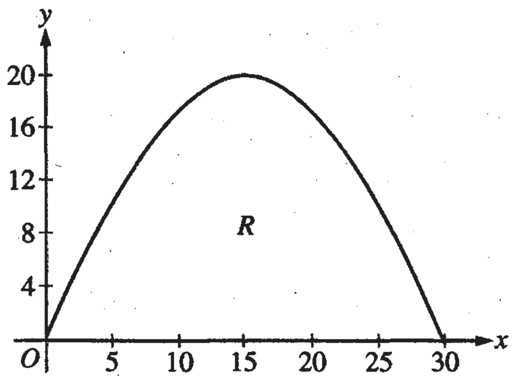
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$A_T = 30 \cdot 20 = 600 \text{ cm}^2$

$A_R = \int_0^{30} 20\sin\left(\frac{\pi x}{30}\right)dx$

$A_R = 381.972 \text{ cm}^2$

The area of the discarded cardboard will be 218.028 cm²

$A_D = A_T - A_R$

$= 600 - 381.972$

$\boxed{A_D = 218.028 \text{ cm}^2}$

Work for problem 1(b)

The cake will contain 300 grams of unsweetened chocolate.

$A = \frac{1}{2}\pi r^2 \qquad r = 20\sin\left(\frac{\pi x}{30}\right)$

$A = \frac{\pi}{2}\left(20\sin\left(\frac{\pi x}{30}\right)\right)^2$

$V = \int_0^{30} A = \frac{\pi}{2}\int_0^{30}\left[20\sin\left(\frac{\pi x}{30}\right)\right]^2 dx$

$V = 6000 \text{ cm}^3$

$6000 \cdot 0.05 = 300$

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Continue problem 1 on page 5.

Work for problem 1(c)

P = arc length + 30

$L = \int_0^{30} \sqrt{\left(\frac{dy}{dx}\right)^2}\, dx$

$L = \int_0^{30} \sqrt{\left(\frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)\right)^2}\, dx$

$L = 65.747$

$P = 65.747 + 30$

$\boxed{P = 95.797 \text{ cm}}$

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Question 1

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: 3 points in part (a), 3 points in part (b), and no points in part (c). In part (c) the student does not have an arclength integral and was not eligible for the answer point.

Sample: 1C
Score: 4

The student earned 4 points: 3 points in part (a), 1 point in part (b), and no points in part (c). In part (a) the student's work is correct. In part (b) the student has an error in the constant factor and earned only 1 of the integral points. The student was eligible for the last point, but the answer is not consistent with the work shown. In part (c) the student does not have an arclength integral and was not eligible for the answer point.

Question 2

A storm washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by $f(t) = \sqrt{t} + \cos t - 3$ meters per hour, t hours after the storm began. The edge of the water was 35 meters from the road when the storm began, and the storm lasted 5 hours. The derivative of $f(t)$ is $f'(t) = \frac{1}{2\sqrt{t}} - \sin t$.

(a) What was the distance between the road and the edge of the water at the end of the storm?

(b) Using correct units, interpret the value $f'(4) = 1.007$ in terms of the distance between the road and the edge of the water.

(c) At what time during the 5 hours of the storm was the distance between the road and the edge of the water decreasing most rapidly? Justify your answer.

(d) After the storm, a machine pumped sand back onto the beach so that the distance between the road and the edge of the water was growing at a rate of $g(p)$ meters per day, where p is the number of days since pumping began. Write an equation involving an integral expression whose solution would give the number of days that sand must be pumped to restore the original distance between the road and the edge of the water.

(a) $35 + \int_0^5 f(t)\,dt = 26.494$ or 26.495 meters	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(b) Four hours after the storm began, the rate of change of the distance between the road and the edge of the water is increasing at a rate of 1.007 $\text{meters}/\text{hours}^2$.	$2: \begin{cases} 1: \text{interpretation of } f'(4) \\ 1: \text{units} \end{cases}$
(c) $f'(t) = 0$ when $t = 0.66187$ and $t = 2.84038$ The minimum of f for $0 \le t \le 5$ may occur at 0, 0.66187, 2.84038, or 5. $f(0) = -2$ $f(0.66187) = -1.39760$ $f(2.84038) = -2.26963$ $f(5) = -0.48027$ The distance between the road and the edge of the water was decreasing most rapidly at time $t = 2.840$ hours after the storm began.	$3: \begin{cases} 1: \text{considers } f'(t) = 0 \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$
(d) $-\int_0^5 f(t)\,dt = \int_0^x g(p)\,dp$	$2: \begin{cases} 1: \text{integral of } g \\ 1: \text{answer} \end{cases}$

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Work for problem 2(a)

$F(0) = 35$

$35 + \int_0^5 F(t)\,dt \approx 35 - 8.505 \approx 26.495$ m

Work for problem 2(b)

$F'(4) = 1.007$

At 4 hours into the thunderstorm, the rate at which the distance between the road and the edge of the water was changing is increasing by 1.007 m/h².

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Continue problem 2 on page 7.

Work for problem 2(c)

$f'(t) = \frac{1}{2\sqrt{t}} - \sin t = 0$

$f'(0.662) = 0$ $\quad \underset{.662}{+ \mid -}$ possible Min

$f'(2.840) = 0$ $\quad \underset{2.840}{- \mid +}$

$f(0) = -2$

$f(2.840) = -2.270$

$f(5) = -0.480$

Decreasing most rapidly at $t = 2.840$

Work for problem 2(d)

$\int_0^5 f(t)\,dt$ distance grown.. ≈ -8.505

$-8.505 + \int_0^t g(p)\,dp = 0$

$\int_0^t g(p)\,dp = 8.505$ m

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Work for problem 2(a)

Let $F(t)$ be the antiderivative of $f(x)$.

$\rightarrow F(t) = \frac{2}{3}t^{\frac{3}{2}} + \sin t - 3t + C$.

Since $F(0) = 35$, $C = 35$.

~~$F(5) = \frac{2}{3}\times 5^{\frac{3}{2}} + \sin(5) - 3\times 5 + C = 35$.~~

~~$C = 43.505$.~~

~~$F(0) = C = \boxed{43.505m}$~~

$F(5) = \frac{2}{3}\times 5^{\frac{3}{2}} + \sin(5) - 3\times 5 + 35$

$= \boxed{26.495m}$

Work for problem 2(b)

$f(t)$ indicates the rate at which the distance between the road and the edge of the water was changing.

Therefore, $f'(t)$ indicates the rate at which the changing rate of the distance changes.

$f'(4) = 1.007$ means that the rate at which the ~~changing~~ changing rate of the distance between the road and the edge of the water is ~~[illegible]~~ $1.007\ m/hr^2$ when the storm lasted for 4 hours.

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Continue problem 2 on page 7.

Work for problem 2(c)

The distance between the road and the edge of the water decreases most rapidly. $\Leftrightarrow$ $f(t)$ is minimum.

$f(t)$ minimum ~~when~~ at the endpoint of $[0,5]$ or at the point at which $f'(t)=0$. ~~and $f''(t)>0$~~.

$f(0)=-3$; $f(5)=-0.480$.

$f'(t)=\frac{1}{2\sqrt{t}}-\sin t=0$. $\rightarrow$ $t=0.662, 2.84$

$f(0.662)=-1.372$.

$f(2.84)=-2.270$.

$\therefore$ minimum at $t=0$ (just when the storm started)

Work for problem 2(d)

The distance that needs to be restored is $35-26.495=8.505$m.

$\rightarrow$ $\boxed{\int_0^x g(p)\,dp = 8.505}$

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Work for problem 2(a)

$\cancel{\frac{dr}{dt}} = f(t) = \sqrt{t} + \cos t - 3$

$0 \le t \le 5$

$f'(t) = \frac{1}{2\sqrt{t}} - \sin t$ $\boxed{d = 35 \quad t = 0}$

(a) $d(5) = ?$ $\int_0^5 f(t)\,dt = d(5) - d(0)$

$= -8.50536.$

$\therefore d(5) = d(0) - 8.505$

$= \boxed{26.495 \text{ m } (3.d.p)}$

Work for problem 2(b)

$f'(4) = 1.007$

$f'(4)$ means that during the fourth hour of the storm, the rate of change of 'the rate of change between the road and the edge of water' was 1.007. i.e.

Continue problem 2 on page 7.

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Work for problem 2(c)

$d =$ distance between road and water

d decreasing most rapidly

$= \frac{d}{dt}$ negative max^m value at $\frac{d}{dt}(d) = f'(t)$

$= f''(t) = 0$.

$\therefore t = 4.68775$ or 42.4106

but $0 \le t \le 5$

$\therefore t = 4.688$ (3.d.p.) hour.

$=$ 4 hours 41 min (nearest whole min) after storm starts.

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Work for problem 2(d)

(d) $g(p)$

Sand lost during storm $= \int_0^5 f(t)\,dt$.

~~Sand pump in~~ $\cancel{\int_0^p g(p)\,dp = \int_0^5 f(t)\,dt}$

~~let $S(p)$ = sand pumped in at time p.~~

$\therefore$ ~~Solution: $S(p)$~~

$\therefore \int_0^5 f(t)\,dt = \int_0^p g(p)\,dp$

for sand to be restored to initial condition.

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Question 2

Sample: 2A
Score: 9

The student earned all 9 points. Note that in part (d) the student's second line earned both points. The t variable that the student uses in the first integral was ignored. That t is in hours after the start of the storm, but the t variable in the student's second integral is in days.

Sample: 2B
Score: 6

The student earned 6 points: 2 points in part (a), 1 point in part (b), 1 point in part (c), and 2 points in part (d). In part (a) the student's work is correct. The student does not include a definite integral but earned the integral point for correct antidifferentiation, use of the initial condition, and evaluation at 5. In part (b) the student earned the units point. Since the response does not include the word "increasing," the interpretation point was not earned. In part (c) the student earned the first point for considering $f'(t) = 0.$ The student did not earn the answer point due to evaluation errors and was not eligible for the justification point. In part (d) the student's boxed equation earned both points.

Sample: 2C
Score: 3

The student earned 3 points: 2 points in part (a), no points in part (b), no points in part (c), and 1 point in part (d). In part (a) the student's work is correct. In part (b) the response does not include the word "increasing" or any units. In part (c) the student is seeking a maximum value rather than a minimum value. The student considers $f''(t) = 0$ instead of $f'(t) = 0.$ In part (d) the student earned the point for the integral of g in spite of using the same name for the upper limit of integration and the variable of integration. The answer point was not earned since the response lacks a negative sign in the integral equation.

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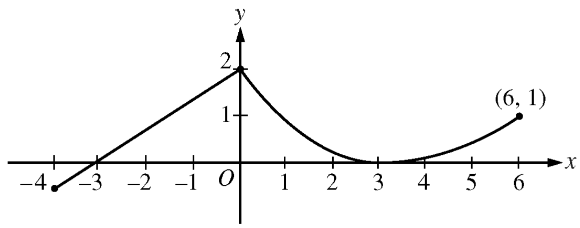
2009 SCORING GUIDELINES (Form B)

Question 3



Graph of f

A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.

(a) Is f differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.

(b) For how many values of a, $-4 \le a < 6$, is the average rate of change of f on the interval $[a, 6]$ equal to 0 ? Give a reason for your answer.

(c) Is there a value of a, $-4 \le a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c, $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.

(d) The function g is defined by $g(x) = \int_0^x f(t)\,dt$ for $-4 \le x \le 6$. On what intervals contained in $[-4, 6]$ is the graph of g concave up? Explain your reasoning.

(a) $\lim\limits_{h \to 0^-} \dfrac{f(h) - f(0)}{h} = \dfrac{2}{3}$

$\lim\limits_{h \to 0^+} \dfrac{f(h) - f(0)}{h} < 0$

Since the one-sided limits do not agree, f is not differentiable at $x = 0$.

$2 : \begin{cases} 1 : \text{sets up difference quotient at } x = 0 \\ 1 : \text{answer with justification} \end{cases}$

(b) $\dfrac{f(6) - f(a)}{6 - a} = 0$ when $f(a) = f(6)$. There are two values of a for which this is true.

$2 : \begin{cases} 1 : \text{expression for average rate of change} \\ 1 : \text{answer with reason} \end{cases}$

(c) Yes, $a = 3$. The function f is differentiable on the interval $3 < x < 6$ and continuous on $3 \le x \le 6$.

Also, $\dfrac{f(6) - f(3)}{6 - 3} = \dfrac{1 - 0}{6 - 3} = \dfrac{1}{3}$.

By the Mean Value Theorem, there is a value c, $3 < c < 6$, such that $f'(c) = \dfrac{1}{3}$.

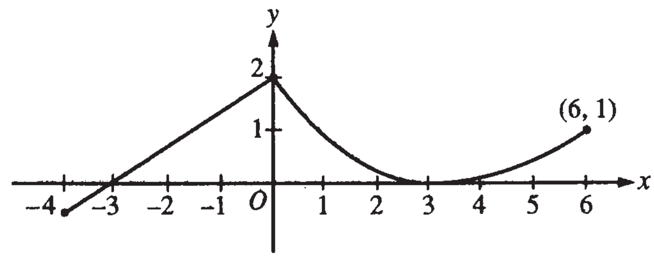
$2 : \begin{cases} 1 : \text{answers “yes” and identifies } a = 3 \\ 1 : \text{justification} \end{cases}$

(d) $g'(x) = f(x)$, $g''(x) = f'(x)$

$g''(x) > 0$ when $f'(x) > 0$

This is true for $-4 < x < 0$ and $3 < x < 6$.

$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{considers } g''(x) > 0 \\ 1 : \text{answer} \end{cases}$



Graph of f

Work for problem 3(a)

f is differentiable at $x=0 \iff \lim\limits_{x\to 0} \dfrac{f(x)-f(0)}{x-0}$ exists

Now $\lim\limits_{x\to 0^+} \dfrac{f(x)-f(0)}{x} < 0$ because $f(x)<f(0)$ at $x>0$

$\lim\limits_{x\to 0^-} \dfrac{f(x)-f(0)}{x} > 0$ because $f(x)<f(0)$ at $x<0$

so $\lim\limits_{x\to 0^+} \dfrac{f(x)-f(0)}{x} \neq \lim\limits_{x\to 0^-} \dfrac{f(x)-f(0)}{x}$

$\Rightarrow \lim\limits_{x\to 0} \dfrac{f(x)-f(0)}{x}$ does not exist $\Rightarrow$ f is not differentiable at $x=0$

Work for problem 3(b)

Average rate of change of f on $[a, 6] = 0$

$\Rightarrow \dfrac{f(6)-f(a)}{6-a} = 0 \Rightarrow f(a) = f(6)$, $a \neq 6$

$\Rightarrow$ # of possible values of a = 2

Continue problem 3 on page 9.

Work for problem 3(c)

Choose $a = 3$.

f is differentiable on $(3,6)$ and continuous on $[3,6]$

By Mean Value Theorem,

$\exists\, c \in [3,6]$ such that $f'(c) = \dfrac{f(6)-f(3)}{6-3}$

$= \dfrac{1-0}{3}$

$= \dfrac{1}{3}$

Thus there is a value of a that satisfies the condition, which is 3.

Work for problem 3(d)

g is concave up on (a,b)

$\Leftrightarrow g''(x) = \frac{d}{dx} g'(x) = \frac{d}{dx} f(x) = f'(x) \geq 0$ on (a,b)

$\Leftrightarrow f(x)$ is increasing on (a,b)

$f(x)$ is increasing on $(-4,0)$ and $(3,6)$

Thus g is concave up on the intervals $(-4,0)$ and $(3,6)$

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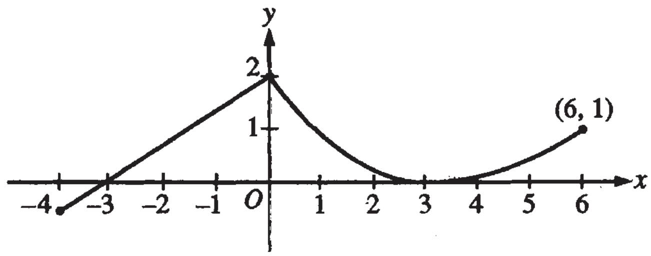
END OF PART A OF SECTION II
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.



Graph of f

Work for problem 3(a)

No, f is not differentiable.

For $f(0)$ to be differentiable, $\lim_{x \to 0^-} f'(x) = \lim_{x \to 0^+} f'(x)$.

$f'(0^-) = \frac{2}{3}$, but $f'(0^+) = 1$.

Work for problem 3(b)

There are two values where the average rate of change of f on $[a,6]$ equals 0. Average rate, or slope of the secant line, must equal to zero: Average rate $= \frac{1 - f(a)}{6 - a}$.

For the slope to be zero, $f(a) = 1$. There are two x values in the graph with a corresponding y value of 1.

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Continue problem 3 on page 9.

Work for problem 3(c)

Yes, there is. For the Mean value theorem, f(x) must be continuous and differentiable at [a, 6]. f(x) with endpoint is continuous and differentiable at points from $x=0$ to $x=6$.

Mean Value Theorem states the following:

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{1}{3}$$

$$= \frac{1-f(a)}{6-a} = \frac{1}{3}$$

At $a=3$, $\frac{1-0}{6-3} = \frac{1}{3}$.

Work for problem 3(d)

For g(x) to be concave up, $g''(x) > 0$.

$g''(x) = f'(x) > 0$.

$f'(x) > 0$ on the intervals $[-4, 2]$ and $[3, 6]$.

END OF PART A OF SECTION II
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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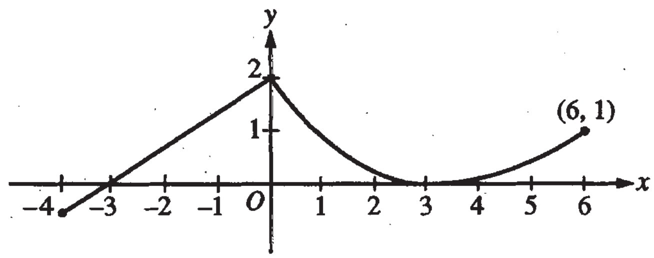

Graph of f

Work for problem 3(a)

No. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \Rightarrow$ nonexistent.

Work for problem 3(b)

$$\frac{\int_a^6 f'(x)\,dx}{6-a} = \frac{f(6)-f(a)}{6-a} = 0.$$

$f(6)=f(a)=1$, $a\neq 6$

$\therefore$ 2.

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Continue problem 3 on page 9.

Work for problem 3(c)

$$\frac{f(6)-f(a)}{6-a} = f'(c) = \frac{1}{3}$$

Yes, f is differentiable at all points of $0 < x < 6$. $-4 < x < 0$.

$\therefore$ There exists a "c" ~~that~~ at which point $f'(c) = \frac{1}{3}$

Work for problem 3(d)

$g''(x) > 0$

$g'(x) = f(x)$

$g''(x) = f'(x)$

$f'(x) > 0$.

$\therefore -3 \leq x < 0,\quad 3 < x \leq 6$.

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END OF PART A OF SECTION II
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Question 3

Sample: 3A
Score: 9

The student earned all 9 points. Note that in part (c) the student affirms the hypotheses of the Mean Value Theorem, but generally that was not required to earn the second point. In part (d) the student earned the first point implicitly via $g''(x) = f'(x)$.

Sample: 3B
Score: 6

The student earned 6 points: no points in part (a), 2 points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the student is not working with a difference quotient. The answer is correct, but the justification is insufficient. In part (b) the student's work is correct. In part (c) the student earned both points even though the statement that "there exists a c with $3 < c < 6$" is not included *and* the student may be implying that f is differentiable at $x = 0$. In part (d) the student earned the first 2 points. The student implicitly connects g' and f via $g''(x) = f'(x)$. The student makes the common error of using $f(0)$, instead of 0, as the right-hand endpoint of one of the intervals.

Sample: 3C
Score: 4

The student earned 4 points: no points in part (a), 2 points in part (b), no points in part (c), and 2 points in part (d). In part (a) the student is working with a difference quotient but not at $x = 0$. The answer is correct, but the justification is insufficient. In part (b) the student's work is correct. In part (c) the student never identifies $a = 3$. In part (d) the student earned the first 2 points, but the answer is not correct. Note that students were not penalized for including the endpoints in the correct intervals.

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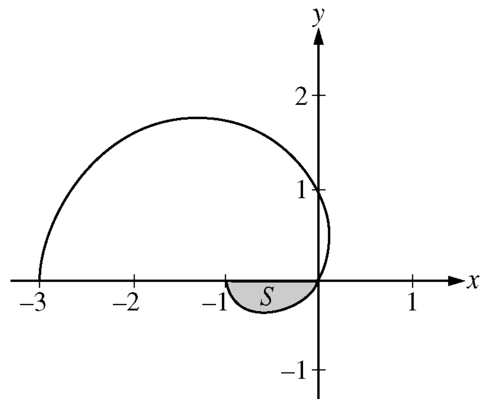
2009 SCORING GUIDELINES (Form B)

Question 4

The graph of the polar curve $r = 1 - 2\cos\theta$ for $0 \le \theta \le \pi$ is shown above. Let S be the shaded region in the third quadrant bounded by the curve and the x-axis.



(a) Write an integral expression for the area of S.

(b) Write expressions for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ.

(c) Write an equation in terms of x and y for the line tangent to the graph of the polar curve at the point where $\theta = \frac{\pi}{2}$. Show the computations that lead to your answer.

(a) $r(0) = -1$; $r(\theta) = 0$ when $\theta = \frac{\pi}{3}$.

Area of $S = \frac{1}{2}\int_0^{\pi/3} (1 - 2\cos\theta)^2 \, d\theta$

$2 : \begin{cases} 1 : \text{limits and constant} \\ 1 : \text{integrand} \end{cases}$

(b) $x = r\cos\theta$ and $y = r\sin\theta$

$$\frac{dr}{d\theta} = 2\sin\theta$$

$$\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta = 4\sin\theta\cos\theta - \sin\theta$$

$$\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta = 2\sin^2\theta + (1 - 2\cos\theta)\cos\theta$$

$4 : \begin{cases} 1 : \text{uses } x = r\cos\theta \text{ and } y = r\sin\theta \\ 1 : \frac{dr}{d\theta} \\ 2 : \text{answer} \end{cases}$

(c) When $\theta = \frac{\pi}{2}$, we have $x = 0$, $y = 1$.

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=\frac{\pi}{2}} = -2$$

The tangent line is given by $y = 1 - 2x$.

$3 : \begin{cases} 1 : \text{values for } x \text{ and } y \\ 1 : \text{expression for } \frac{dy}{dx} \\ 1 : \text{tangent line equation} \end{cases}$

NO CALCULATOR ALLOWED

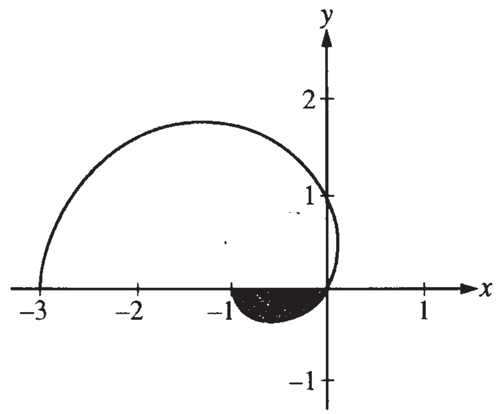
CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

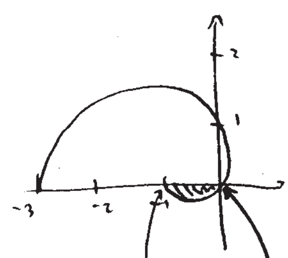
No calculator is allowed for these problems.



Work for problem 4(a)

$$\int_0^{\frac{1}{3}\pi} \frac{1}{2} r^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{\frac{1}{3}\pi} (1-2\cos\theta)^2 \, d\theta$$



$r=-1$ $\quad$ $r=0$.

$1-2\cos\theta=-1$ $\quad$ $\therefore 1-2\cos\theta=0$

$\cos\theta=1$ $\quad$ $\cos\theta=\frac{1}{2}$

$\therefore \theta=0$, $\quad$ $\theta=\frac{1}{3}\pi$

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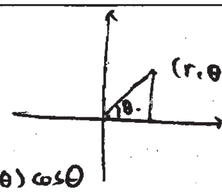
Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(b)



$r = f(\theta)$

$(r, \theta) \Rightarrow (r\cos\theta,\ r\sin\theta)$
$= (f(\theta)\cos\theta,\ f(\theta)\sin\theta)$

$r = 1 - 2\cos\theta = f(\theta)$

$x = r \cdot \cos\theta = f(\theta)\cos\theta$

$y = r \cdot \sin\theta = f(\theta)\sin\theta$

$f(\theta) = 1 - 2\cos\theta \quad f'(\theta) = 2\sin\theta$

$$\frac{dx}{d\theta} = f'(\theta)\cos\theta - f(\theta)\sin\theta = 2\sin\theta\cos\theta - (1-2\cos\theta)\sin\theta$$
$$= 2\sin 2\theta - \sin\theta \qquad (\because 2\sin\theta\cos\theta = \sin 2\theta)$$

$$\frac{dy}{d\theta} = f'(\theta)\sin\theta + f(\theta)\cos\theta = 2\sin^2\theta + (1-2\cos\theta)\cos\theta$$
$$= 2\sin^2\theta + \cos\theta - 2\cos^2\theta$$

Work for problem 4(c)

$\theta = \frac{\pi}{2}$ $\left(\begin{array}{l} x = r\cdot\cos\theta = (1-2\cos\theta)\cos\theta \\ \quad = 0 \\ y = r\cdot\sin\theta = (1-2\cos\theta)\sin\theta \\ \quad = 1 \end{array} \right.$

line tangent:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2\sin^2\theta + \cos\theta - 2\cos^2\theta}{2\sin 2\theta - \sin\theta}.$$

$$\therefore \text{ where } \theta = \frac{\pi}{2} \qquad \frac{dy}{dx} = \frac{2\sin^2(\frac{\pi}{2}) + \cos\frac{\pi}{2} - 2\cos^2(\frac{\pi}{2})}{2\sin\pi - \sin\frac{\pi}{2}} = \frac{2}{-1} = -2.$$

$\therefore$ line tangent : $y - 1 = -2(x - 0)$

$\Rightarrow \quad y = -2x + 1$

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 4B1

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CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

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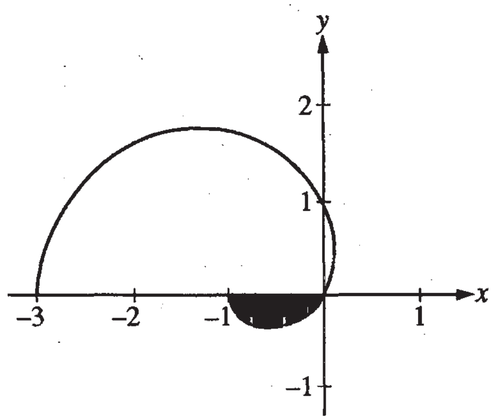

Work for problem 4(a)

$1-2\cos\theta = 0 \quad \cos\theta = \frac{1}{2} \quad \theta = \frac{\pi}{6}$

$1-2\cos\theta = -1 \quad \cos\theta = 1 \quad \theta = 0.$

$S = \frac{1}{2}\int_0^{\frac{\pi}{6}} r^2\, d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{6}} (1-2\cos\theta)^2\, d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{6}} (1-4\cos\theta + 4\cos^2\theta)\, d\theta.$

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Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(b)

$x = r\cos\theta$

$= (1-2\cos\theta)\cos\theta$

$= \cos\theta - 2\cos^2\theta$

$2\cos^2\theta = \cos 2\theta + 1$

$\frac{dx}{d\theta} = -\sin\theta + 2\sin 2\theta$

$y = r\sin\theta$

$= (1-2\cos\theta)\sin\theta$

$= \sin\theta - \sin 2\theta$

$\frac{dy}{d\theta} = \cos\theta - 2\cos 2\theta$

Work for problem 4(c)

$r = 1 - 2\cos\frac{\pi}{2} = 1$

when $\theta = \frac{\pi}{2}$ the coordinate of the point is $(0, 1)$

the coordinate of the center of the circle that pass through point $(0,1)$ and $(0,0)$ is $(-\sqrt{2}, 0)$.

$(x+\sqrt{2})^2 + (y+\sqrt{2})^2 = 1$

$x^2 + 2\sqrt{2}x + 2 + y^2 + 2\sqrt{2}y + 2 = 1$

$x^2 + y^2 + 2\sqrt{2}(x+y) = -3$

$2x + 2y\frac{dy}{dx} + 2\sqrt{2} + 2\sqrt{2}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{2x+2\sqrt{2}}{2y+2\sqrt{2}} = -\frac{2\sqrt{2}}{2+2\sqrt{2}}$

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CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

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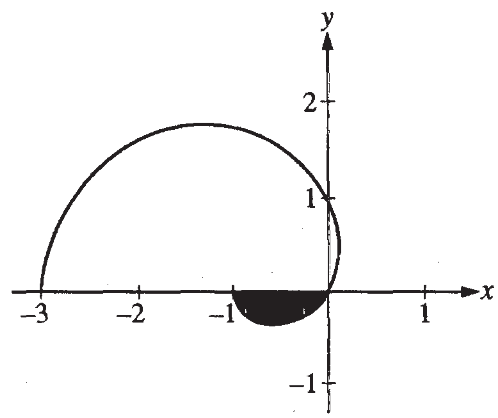

Work for problem 4(a)

$1-2\cos\theta=0$ when $\cos\theta=\frac{1}{2}$. $\therefore \theta=\frac{\pi}{4}$ when $r=0$.

~~Area~~ $\int_0^{\frac{\pi}{4}}(1-2\cos\theta)\,d\theta$ is negative though, because the graph is under the x-axis.

therefore, $S=-\int_0^{\frac{\pi}{4}}(1-2\cos\theta)\,d\theta$

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Continue problem 4 on page 11.

Work for problem 4(b)

$x = r\cos\theta$

$\therefore \frac{dx}{d\theta} = -r\sin\theta$

$y = r\sin\theta$

$\therefore \frac{dy}{d\theta} = r\cos\theta$

Work for problem 4(c)

~~tangent line :~~

when $\theta = \frac{\pi}{2}$, $\cos\theta = 0$.

$\therefore r = 1$.

$\therefore (x, y) = (0, 1)$.

$\therefore$ tangent line : $(y-1) = \frac{dy}{dx}(x-0)$.

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{r\cos\theta}{-r\sin\theta} = -\cot\theta$.

$\therefore y-1 = -\cot\theta \cdot x$

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Question 4

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 1 point in part (a), 4 points in part (b), and 1 point in part (c). In part (a) the student earned the integrand point, but the student's limits are incorrect. In part (b) the student's work is correct. Prior to differentiating, the student uses a trigonometric identity to rewrite the expression for x in terms of θ. Although $\frac{dr}{d\theta}$ is not explicitly stated, the student earned the $\frac{dr}{d\theta}$ point. In part (c) the student earned the first point for the coordinates of the point of tangency.

Sample: 4C
Score: 3

The student earned 3 points: no points in part (a), 1 point in (b), and 2 points in part (c). In part (a) the student's work is incorrect. In part (b) the student earned the first point. In part (c) the student's third line earned the first point. The second point is conceptual. The student earned the point by importing incorrect derivatives from part (b) and combining them correctly to form $\frac{dy}{dx}$. The student did not earn the third point in part (c).

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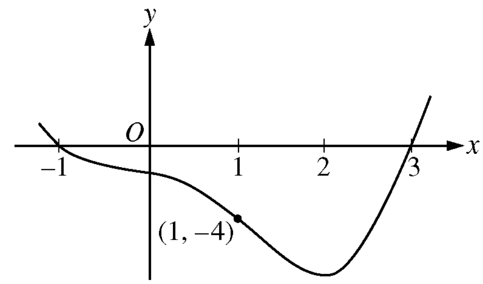
Question 5

Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.



Graph of f'

(a) Write an equation for the line tangent to the graph of g at $x = 1$.

(b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

(c) The second derivative of g is $g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.

(d) Find the average rate of change of g', the derivative of g, over the interval $[1, 3]$.

(a) $g(1) = e^{f(1)} = e^2$

$g'(x) = e^{f(x)} f'(x), \quad g'(1) = e^{f(1)} f'(1) = -4e^2$

The tangent line is given by $y = e^2 - 4e^2(x - 1)$.

$$3 : \begin{cases} 1 : g'(x) \\ 1 : g(1) \text{ and } g'(1) \\ 1 : \text{tangent line equation} \end{cases}$$

(b) $g'(x) = e^{f(x)} f'(x)$

$e^{f(x)} > 0$ for all x

So, g' changes from positive to negative only when f' changes from positive to negative. This occurs at $x = -1$ only. Thus, g has a local maximum at $x = -1$.

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

(c) $g''(-1) = e^{f(-1)}\left[(f'(-1))^2 + f''(-1)\right]$

$e^{f(-1)} > 0$ and $f'(-1) = 0$

Since f' is decreasing on a neighborhood of -1, $f''(-1) < 0$. Therefore, $g''(-1) < 0$.

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

(d) $\dfrac{g'(3) - g'(1)}{3 - 1} = \dfrac{e^{f(3)} f'(3) - e^{f(1)} f'(1)}{2} = 2e^2$

$$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer} \end{cases}$$

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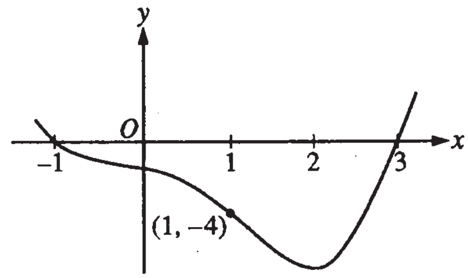

Graph of f'

Work for problem 5(a)

$g'(1) = e^{f(1)} \cdot f'(1) = -4e^2$

$e^2 = -4e^2 + C$

$\Rightarrow C = 5e^2$

Equation of tangent line to g at $x = 1$:

$y = -4e^2 x + 5e^2$ //

Work for problem 5(b)

g has a local maximum when g' changes sign from positive to negative.

$g'(x) = e^{f(x)} \cdot f'(x)$

$e^{f(x)}$ is always positive, $\therefore$ $g'(x)$ changes sign from positive to negative when $f'(x)$ does so.

$f'(x)$ changes sign from positive to negative at $x = -1$,

$\therefore$ g has a local maximum at $x = -1$. //

Continue problem 5 on page 13.

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NO CALCULATOR ALLOWED

Work for problem 5(c)

$g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right]$

$e^{f(x)}$ is always positive

$(f'(-1))^2 = 0$

$f''(-1)$ is negative.

$\therefore g''(-1)$ is negative.

Work for problem 5(d)

$g'(x) = e^{f(x)} \cdot f'(x)$

$g'(1) = e^{f(1)} \cdot f'(1) = -4e^2$

$g'(3) = e^{f(3)} \cdot f'(3) = 0$

Average rate of change $= \dfrac{0-(-4e^2)}{3-1}$

$= 2e^2$

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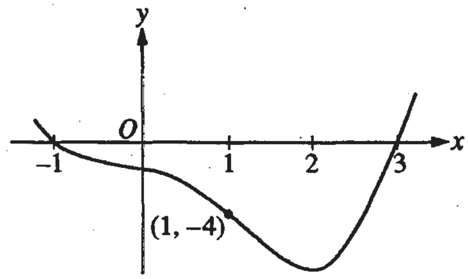

Graph of f'

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Work for problem 5(a)

$g'(x) = e^{f(x)} \cdot f'(x)$

$g'(1) = e^{f(1)} \cdot f'(1) = e^2 \cdot -4 = -4e^2$

$g(1) = e^{f(1)} = e^2$

$\boxed{y - e^2 = -4e^2 (x-1)}$

Work for problem 5(b)

$g'(x) = e^{f(x)} \cdot f'(x)$

$e^{f(x)} \Rightarrow$ always positive

g has a local maximum at $x = -1$ because $g'(x)$ changes from positive to negative.

Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

$g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x) \right]$

$g''(-1)$ is negative, $e^{f(x)}$ is positive because any ~~e~~ raised to any number is positive, $f'(-1) = 0$ (given) and $f''(-1) < 0$ (from the graph), so $g''(-1)$ is a positive $*$ (zero + negative) which comes out to be a negative value,

Work for problem 5(d)

average rate of change of $g' = \frac{1}{3-1} \int_1^3 g''(x)\,dx$

$= \frac{1}{3-1} \cdot g'(x) \Big|_1^3 = \frac{g'(3) - g'(1)}{2}$

$g'(3) = e^{f(3)} \cdot f'(3) = e^{f(3)} \cdot 0 = 0$

$g'(1) = -4e^2$ (from 5(a))

$= \frac{0 - (-4e^2)}{2} = \frac{4e^2}{2} = \boxed{2e^2}$

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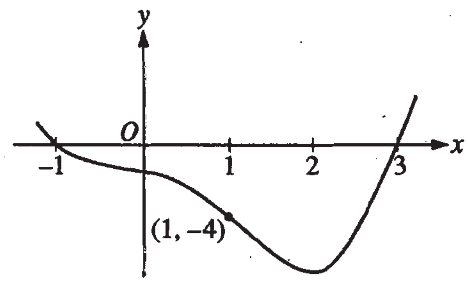

Graph of f'

Work for problem 5(a)

$g'(x) = e^{f(x)} \cdot f'(x)$

$g'(1) = e^{2} \cdot (-4)$

$= -4e^{2}$

$(y - e) = -4e^{2}(x-1)$

Work for problem 5(b)

$f(x)$ has local max at $x=-1$

$g(x) = e^{f(x)}$

$\therefore g(x)$ has local max at $x=-1$

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Continue problem 5 on page 13.

Work for problem 5(c)

$f'(-1) = 0$

$f''(-1) < 0$ since $f'(x)$ is decreasing from $[-1/2, 2]$

since $f(1) = 2$ and $f(x)$ has only decreased from $f(-1)$ to $f(1)$, $f(-1) > 0$

$g''(x) = e^{f(x)}(0 + f''(x)$

$\therefore g''(-1) < 0$

Work for problem 5(d)

$g'(x) = e^{f(x)} \cdot f'(x)$

$g(1) = e^2 \cdot -4$

$= -4e^2$

$g(3) = 0$

avg rate of change $= \dfrac{g(3) - g(0)}{3 - 0}$

$= \dfrac{0 + 4e^2}{3}$

$= \dfrac{4e^2}{3}$

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Question 5

Sample: 5A
Score: 9

The student earned all 9 points. Note that in part (a) the student's first line earned the point for $g'(x)$. The student includes $g(1)$ implicitly in the second equation. In part (c) the justification is sufficient although the student does not explain why $f''(-1)$ is negative.

Sample: 5B
Score: 6

The student earned 6 points: 3 points in part (a), 1 point in part (b), 2 points in part (c), and no points in part (d). In part (a) the student's work is correct. In part (b) the student earned the answer point, but the justification is insufficient. The student does not describe the sign change in g'. In part (c) the student's work is correct. In part (d) the student is not working with the correct difference quotient.

Sample: 5C
Score: 3

The student earned 3 points: 1 point in part (a), 1 point in part (b), 1 point in part (c), and no points in part (d). In part (a) the student earned the point for $g'(x)$. The student does not have a value for $g(1)$. As a result, the second point was not earned, and the student was not eligible for the third point. In parts (b) and (c) the student earned the answer points. Both justifications are insufficient. In part (d) the student is not working with the correct difference quotient.

Question 6

The function f is defined by the power series

$$f(x) = 1 + (x+1) + (x+1)^2 + \cdots + (x+1)^n + \cdots = \sum_{n=0}^{\infty} (x+1)^n$$

for all real numbers x for which the series converges.

(a) Find the interval of convergence of the power series for f. Justify your answer.

(b) The power series above is the Taylor series for f about $x = -1$. Find the sum of the series for f.

(c) Let g be the function defined by $g(x) = \int_{-1}^{x} f(t)\,dt$. Find the value of $g\left(-\frac{1}{2}\right)$, if it exists, or explain why $g\left(-\frac{1}{2}\right)$ cannot be determined.

(d) Let h be the function defined by $h(x) = f\left(x^2 - 1\right)$. Find the first three nonzero terms and the general term of the Taylor series for h about $x = 0$, and find the value of $h\left(\frac{1}{2}\right)$.

(a) The power series is geometric with ratio $(x+1)$.
The series converges if and only if $|x+1| < 1$.
Therefore, the interval of convergence is $-2 < x < 0$.

$$3 : \begin{cases} 1 : \text{identifies as geometric} \\ 1 : |x+1| < 1 \\ 1 : \text{interval of convergence} \end{cases}$$

OR

$$\lim_{n\to\infty} \left| \frac{(x+1)^{n+1}}{(x+1)^n} \right| = |x+1| < 1 \text{ when } -2 < x < 0$$

At $x = -2$, the series is $\sum_{n=0}^{\infty} (-1)^n$, which diverges since the terms do not converge to 0. At $x = 0$, the series is $\sum_{n=0}^{\infty} 1$, which similarly diverges. Therefore, the interval of convergence is $-2 < x < 0$.

OR

$$3 : \begin{cases} 1 : \text{sets up limit of ratio} \\ 1 : \text{radius of convergence} \\ 1 : \text{interval of convergence} \end{cases}$$

(b) Since the series is geometric,

$$f(x) = \sum_{n=0}^{\infty} (x+1)^n = \frac{1}{1-(x+1)} = -\frac{1}{x} \text{ for } -2 < x < 0.$$

1 : answer

(c) $g\left(-\frac{1}{2}\right) = \int_{-1}^{-\frac{1}{2}} -\frac{1}{x}\,dx = -\ln|x| \Big|_{x=-1}^{x=-\frac{1}{2}} = \ln 2$

$$2 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{value} \end{cases}$$

(d) $h(x) = f\left(x^2 - 1\right) = 1 + x^2 + x^4 + \cdots + x^{2n} + \cdots$

$h\left(\frac{1}{2}\right) = f\left(-\frac{3}{4}\right) = \frac{4}{3}$

$$3 : \begin{cases} 1 : \text{first three terms} \\ 1 : \text{general term} \\ 1 : \text{value of } h\left(\frac{1}{2}\right) \end{cases}$$

NO CALCULATOR ALLOWED

Work for problem 6(a)

By the Ratio Test, $\lim_{n\to\infty}\left|\frac{(x+1)^{n+1}}{(x+1)^n}\right| = |x+1| < 1.$

or $-2<x<0$. The radius of convergence is 1.

Now consider both endpoints.

When $x=-2$, $f(x) = 1-1+1-1+\cdots$, which diverges;

when $x=0$, $f(x)=1+1+1+\cdots$, which also diverges.

Thus, the interval of convergence of f is

$(-2, 0)$.

Work for problem 6(b)

$$f(x) = 1+(x+1)+(x+1)^2+\cdots+(x+1)^n+\cdots$$

$$= \frac{1}{1-(x+1)}$$

$$= -\frac{1}{x}$$

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Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$g(-\frac{1}{2}) = \int_{-1}^{-\frac{1}{2}} f(t)\,dt = -\int_{-1}^{-\frac{1}{2}} \frac{dt}{t} = -\ln|t|\Big|_{-1}^{-\frac{1}{2}} = \ln 2$

and $-\frac{1}{2}$ is within the interval of convergence.

Work for problem 6(d)

$h(x) = f(x^2-1) = 1 + x^2 + x^4 + \cdots + x^{2n} + \cdots$

$h(\frac{1}{2}) = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$

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Work for problem 6(a)

~~x ● -1 ○~~

$-1 < x+1 < 1$ (in order to converge $\sum_{n=1}^{\infty}(x+1)^n$, $x+1$ has to be between -1 and 1.)

$\therefore \underline{-2 < x < 0}$

Work for problem 6(b)

$f(x) = 1 + (x+1) + (x+1)^2 + \cdots + (x+1)^n$

$= 1 \times \frac{1}{1-(x+1)} = \frac{1}{-x}$

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Work for problem 6(c)

$\int_{-1}^{x} f(t)\,dt = g(x)$

$= x + \frac{(x+1)^2}{2} + \frac{(x+1)^3}{3} + \cdots$

$g(-\frac{1}{2}) = -\frac{1}{2} + \frac{(\frac{1}{2})^2}{2} + \frac{(\frac{1}{2})^3}{3} + \cdots$

it can't be determined, because the ~~general term~~ $g(x)$ is $\sum_{n=1}^{\infty} (\frac{1}{2})^{n+1} \frac{1}{(n+1)}$ which does not converge.

Work for problem 6(d)

$h(x) = f(x^2 - 1)$

$= 1 + (x^2) + (x^2)^2 + \cdots + (x^2)^n$

$h(\frac{1}{2}) = 1 + (\frac{1}{2})^2 + (\frac{1}{2})^4 + (\frac{1}{2})^6 + \cdots$

$= 1 \times \frac{1}{1 - \frac{1}{4}} = 1 \times \frac{4}{3} = \frac{4}{3}$

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Work for problem 6(a)

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(x+1)^n \cdot (x+1)}{(x+1)^n}\right| = |x+1| < 1$$

$-1 < x+1 < 1$

$-2 < x < 0$

At $x=0$,

$f(x) = \sum_{n=0}^{\infty} 1^n \rightarrow$ diverges.

At $x=-2$,

$f(x) = \sum_{n=0}^{\infty} (-1)^n \rightarrow$ diverges.

$\therefore$ the interval of convergence is $-2<x<0$

Work for problem 6(b)

The series is a geometric sequence.

Because the series converges,

$(x+1) < 1$.

Therefore, the sum of the series is $\frac{a}{1-r} = \frac{1}{1-(x+1)} = \frac{1}{1-x-1}$

$= \frac{1}{-x}$.

At $x=-1$,

the sum is 1.

$\therefore$ 1.

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Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$g(-\frac{1}{2})$ does not exists.

Because $g(x) = \ln(\frac{1}{2}+x)$.

When $x=-\frac{1}{2}$, $g(x) = \ln(0)$, so the value doesn't exist.

Work for problem 6(d)

$P_3(x) = h(x) + h'(x)\times x + \frac{h''(x)\times x^2}{2!}$

$= f(x^2-1) + f'(x^2-1)\times 2x^2 + 4\frac{f''(x^2-1)(x^2+1)x^2}{2}$

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Question 6

Sample: 6A
Score: 9

The student earned all 9 points. Note that in part (b) it was not necessary for students to explain their reasoning beyond using the formula for the sum of a convergent geometric series.

Sample: 6B
Score: 6

The student earned 6 points: 2 points in part (a), 1 point in part (b), no points in part (c), and 3 points in part (d). In part (a) the student did not earn the first point since the series is not identified as geometric. In part (b) the student's work is correct and was sufficient to earn the point. In part (c) the student did not earn any points. The student attempts to work with the series for $f(t)$ instead of the closed form expression $-\frac{1}{t}$. (The student would have been eligible for the first point using this method if the displayed antiderivative terms were all correct and included a correct general term.) In part (d) the student's work is correct.

Sample: 6C
Score: 3

The student earned 3 points: 2 points in part (a), 1 point in part (b), no points in part (c), and no points in part (d). In part (a) the student earned the second and third points, using the first method in the scoring guidelines. The series is not identified as geometric. In part (b) the student's work is correct and was sufficient to earn the point. The additional statement concerning the sum at $x = -1$ was ignored.